Steel Beam Moment Strength

Flexural strength of a steel wide-flange beam section.

Assumptions

[ASSUME] AISC 14th Edition controls design

[ASSUME] Beam web is unstiffened

Inputs

Beam ultimate moment demand;	$M_u = 30\ \mathrm{kip - ft}$
Beam unbraced length;	$L_b = 20\ \mathrm{ft}$
Beam section size;	$\mathrm{section} = \mathrm{W18X40}$
Steel yield strength;	$F_y = 50\ \mathrm{ksi}$
Steel ultimate strength;	$F_u = 65\ \mathrm{ksi}$
Modulus of elasticity;	$E = 29000\ \mathrm{ksi}$
Lateral-torsional buckling modification factor;	$C_b = 1$

Section Properties

$b = 6.02\ \mathrm{in}$

$d = 17.9\ \mathrm{in}$

$S_x = 68.4\ \mathrm{in}^3$

$Z_x = 78.4\ \mathrm{in}^3$

$r_y = 1.27\ \mathrm{in}$

$r_{ts} = 1.56\ \mathrm{in}$

$J = 0.81\ \mathrm{in}^4$

$h_o = 17.4\ \mathrm{in}$

$b_f/2t_f = 5.73$

$h/t_w = 50.9$

1. Beam Flexural Capacity

Flexural resistance factor:

$\phi_b = 0.9$ [AISC F1(1)]

1.1. Section Compactness

[AISC Table B4.1b(10)]

$$\lambda_{pf} = 0.38 \cdot \sqrt{\frac{\mathrm{e}}{F_y}} = 0.38 \ \cdot \sqrt{\frac{2.718}{50 \text{ ksi}}}$$

$$\therefore \lambda_{pf} = 0.0886$$

$$Check\ b_f/2t_f \ \leq \ \lambda_{pf}$$

$$5.73 \ \leq \ 0.0886$$

$$\therefore ERROR : NotCompactFlange$$

[AISC Table B4.1b(15)]

$$\lambda_{pw} = 3.76 \cdot \sqrt{\frac{\mathrm{e}}{F_y}} = 3.76 \ \cdot \sqrt{\frac{2.718}{50 \text{ ksi}}}$$

$$\therefore \lambda_{pw} = 0.8767$$

$$Check\ h/t_w \ \leq \ \lambda_{pw}$$

$$50.9 \ \leq \ 0.8767$$

$$\therefore ERROR : NotCompactWeb$$

1.2. Plastic Moment Strength

Nominal plastic moment strength:

[AISC Eq. F2-1]

$$M_p = \frac{F_y \cdot Z_x}{12 \text{ in/ft}} = \frac{50 \text{ ksi} \cdot 78.4 \text{ in}^3}{12 \text{ in/ft}}$$

$$\therefore M_p = 326.7 \text{ kip} - \text{ft}$$

1.3. Yielding Strength

[AISC Eq. F2-1]

$$M_{ny} = M_p = 326.7$$

$$\therefore M_{ny} = 326.7 \text{ kip} - \text{ft}$$

1.4. Lateral-Torsional Buckling

[AISC Eq. F2-5]

$$L_p = \frac{1.76 \cdot r_y \cdot \sqrt{\frac{\mathrm{e}}{F_y}}}{12 \text{ in/ft}}$$

$$= \frac{1.76 \ \cdot 1.27 \text{ in} \cdot \sqrt{\frac{2.718}{50 \text{ ksi}}}}{12 \text{ in/ft}}$$

$$\therefore L_p = 0.04343 \text{ ft}$$

$c = 1$ [AISC Eq. F2-8a]

[AISC Eq. F2-6]

$$L_r = \frac{\frac{1.95 \cdot r_{ts}}{12 \text{ in/ft}} \cdot E}{0.7 \cdot F_y} \cdot \sqrt{\frac{J \cdot c}{S_x \cdot h_o} + \sqrt{\left(\frac{J \cdot c}{S_x \cdot h_o}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot F_y}{\mathrm{e}}\right)^2}}$$

$$= \frac{\frac{1.95 \ \cdot 1.56 \text{ in}}{12 \text{ in/ft}} \cdot 29000 \text{ ksi}}{0.7 \ \cdot 50 \text{ ksi}} \cdot \sqrt{\frac{0.81 \text{ in}^4 \cdot 1}{68.4 \text{ in}^3 \cdot 17.4 \text{ in}} + \sqrt{\left(\frac{0.81 \text{ in}^4 \cdot 1}{68.4 \text{ in}^3 \cdot 17.4 \text{ in}}\right)^2 + 6.76 \ \cdot \left(\frac{0.7 \ \cdot 50 \text{ ksi}}{2.718}\right)^2}}$$

$$\therefore L_r = 1215 \text{ ft}$$

$$\rightarrow L_p \ < \ L_b \ \leq \ L_r$$

[AISC F2.2(b)]

$$M_{ncr} = C_b \cdot \left(M_p - \frac{\left(M_p - \frac{0.7 \cdot F_y \cdot S_x}{12 \text{ in/ft}}\right) \cdot L_b - L_p}{L_r - L_p}\right)$$

$$= 1 \ \cdot \left(326.7 \text{ kip} - \text{ft} - \frac{\left(326.7 \text{ kip} - \text{ft} - \frac{0.7 \ \cdot 50 \text{ ksi} \cdot 68.4 \text{ in}^3}{12 \text{ in/ft}}\right) \cdot 20 \text{ ft} - 0.04343 \text{ ft}}{1215 \text{ ft} - 0.04343 \text{ ft}}\right)$$

$$\therefore M_{ncr} = 324.6 \text{ kip} - \text{ft}$$

[AISC Eq. F2-2]

$$M_{nltb} = \min\left(M_{ncr}, M_p\right) = \min\left(324.6 \text{ kip} - \text{ft}, 326.7 \text{ kip} - \text{ft}\right)$$

$$\therefore M_{nltb} = 324.6 \text{ kip} - \text{ft}$$

1.5. Controlling Strength

Design flexural strength of the section:

$$\phi M_n = \phi_b \cdot \min\left(M_{ny}, M_{nltb}\right) = 0.9 \ \cdot \min\left(326.7 \text{ kip} - \text{ft}, 324.6 \text{ kip} - \text{ft}\right)$$

$$\therefore \phi M_n = 292.1 \text{ kip} - \text{ft}$$

$$Check \ M_u \ \leq \ \phi M_n$$

$$30 \text{ kip} - \text{ft} \ \leq \ 292.1 \text{ kip} - \text{ft}$$

$$\therefore OK$$